

# Pulse propagation in one-dimensional disordered photonic crystals: Interplay of disorder with instantaneous and relaxing nonlinearities

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Propagation of ultrashort light pulses in disordered multilayers is studied by using numerical simulations in time domain. We consider cases of instantaneous and noninstantaneous Kerr nonlinearities of the structure materials. The competitive nature of disorder and nonlinearity is revealed on the long and short timescales. We also pay special attention to the effect of pulse self-trapping in the photonic crystal with relaxing nonlinearity and show the dependence of this effect on the level of disorder. We believe that the results reported here will be useful not only in the field of optics but also from the standpoint of the general problem of classical waves propagation in nonlinear disordered periodic media.

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## 1. Introduction

Disordered photonic structures have become the object of active study in recent years. Such interest is substantially due to observation of new optical effects such as the Anderson localization of light [1, 2], coherent backscattering in the low disorder limit [3] and deviations of statistical properties of light from the classical diffusion picture (subdiffusion [4] or superdiffusion [5, 6]). Traditionally, the realization of optical disordered system is the strongly scattering medium, such as suspension of dielectric particles or specially prepared glass. Last years, more technologically complex systems attract attention, for example, the “purposely spoiled” photonic crystals [7–9], microstructured waveguides [10], and metamaterials [11]. The study of such artificial disordered structures is not only intriguing in itself, but also has great practical value, since the products of real-life nanofabrication are never perfect.

Introduction of nonlinearity to disordered systems strongly enhances the complexity and richness of possible optical dynamics. Though there are many works devoted to this topic, one cannot say that the problem of disorder/nonlinearity interaction is fully understood. The results obtained to date include suppression of the Anderson localization in photonic lattices [4, 12], bistability of light transmission through nonlinear random

media due to resonant localized modes [13], instability of waves in random media with instantaneous and noninstantaneous Kerr nonlinearity [14, 15], influence of nonlinearity on random laser output [16], etc. Since in this paper we deal with pulsed radiation propagation, further some results on this particular topic are briefly discussed.

One of the main features of pulse transmission which allows to make conclusions on light diffusion and localization is the “tail” of the pulse, i.e. the shape of intensity decrease at the output of the medium. Exponential tail is the indication of classical diffusion, while nonexponentiality may evidence energy transfer to long-living states and, hence, Anderson localization. Appearance of the nonexponential tail induced by nonlinearity in a three-dimensional random medium was reported in Ref. [17]. A number of works was dedicated to the influence of the so-called transverse disorder on wave-packet dynamics in two-dimensional nonlinear structures. In particular, it was shown [18] that weak nonlinearity in a one-dimensional disordered lattice can destroy the Anderson localization, so that the initially localized wave packet spreads slower than in the diffusion regime (subdiffusion). On the other hand, Lahini *et al.* [19] demonstrated that the packet localization in a similar model described by the discrete nonlinear Schrödinger equation can be stronger or weaker for different types of modes. This conclusion was confirmed experimentally using the disordered set of coupled optical waveguides. On the basis of numerical simulations of wave-packet

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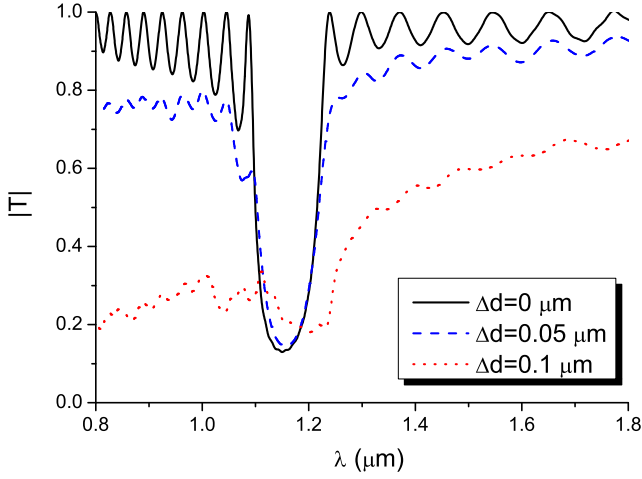


Fig. 1. (Color online) Transmission spectra of the photonic crystal with 10 periods for different values of disorder amplitude. The curves are averaged over 20 realizations.

dynamics in the one-dimensional lattice, the authors of Ref. [20] distinguished three regimes of evolution: (i) weak-nonlinearity regime when the packet stays localized for some time and then spreads subdiffusively, (ii) moderate-nonlinearity regime when the packet experiences subdiffusion from the very outset, (iii) strong-nonlinearity regime when some part of the packet gets localized due to self-trapping. Leaving aside purely nonlinear process of self-trapping, one can say that any level of nonlinearity is enough to destroy the Anderson localization. The effect of self-trapping in disordered systems was studied in later works as well (see, for example, [21, 22]).

Another consequence of interaction between disorder and nonlinearity revealed in Ref. [23] implies formation at certain conditions of the stable states combining the properties of localized modes and solitary waves. It was shown that disorder assists nonlinearity in threshold lowering for generation of such “superlocalized” solitons. It was proposed [24] that pulsed radiation in the form of self-induced transparency solitons can be used to pump localized Anderson states in a disordered two-level medium. Since these Anderson states are similar to the modes of a laser cavity, the scheme described is expected to operate as a peculiar two-level laser. Finally, if the nonlinearity is nonlocal, destabilization of localized states becomes more problematic: the stronger nonlocality, the higher power needed to obtain mode instability [25].

In this paper, we consider ultrashort pulse propagation in a one-dimensional disordered multilayer (photonic crystal) with instantaneous and noninstantaneous nonlinearity. In other words, we deal not with *transverse*, but with *longitudinal* disorder. Theoretical study of perfect multilayer systems and their applications continues for many years (see, for example, Refs. [26, 27]). It is known that there is no threshold for the Anderson

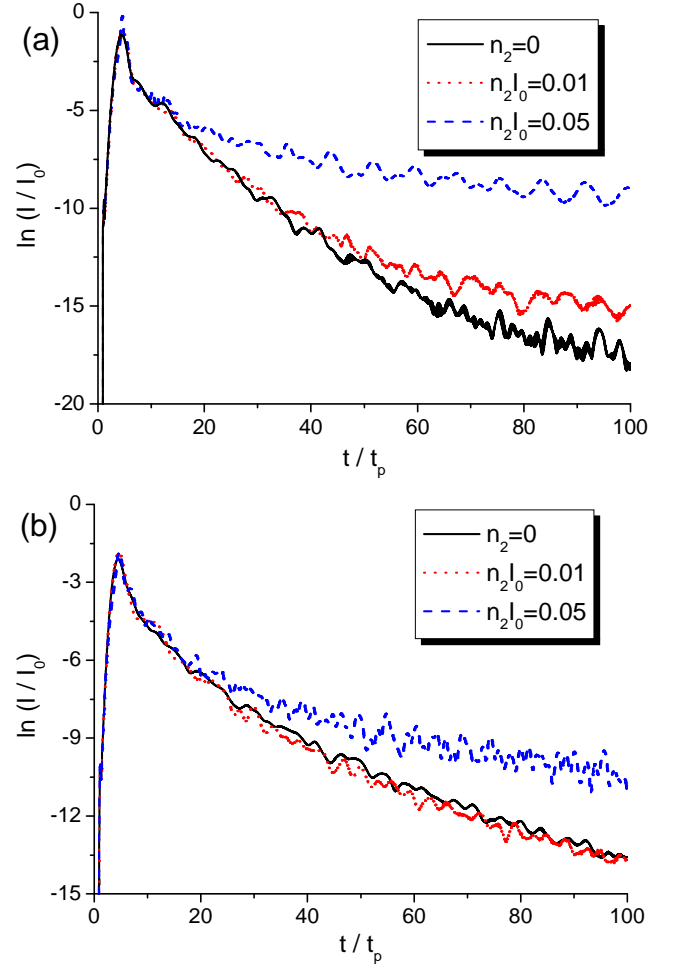


Fig. 2. (Color online) Dynamics of transmitted intensity logarithm for the photonic crystal with 50 periods at the disorder amplitudes (a)  $\Delta d = 0.05 \mu\text{m}$ , (b)  $\Delta d = 0.1 \mu\text{m}$ . The curves are averaged over 25 realizations.

localization in one-dimensional random media. The peculiarity of photonic crystals is their periodicity which results in different regimes of localization [28]. We are interested in the regime of both *strong disorder*, when the range of Bragg-like reflection is effectively extended beyond the band gap, and *strong nonlinearity*, when the effects of pulse shape transformation occur. In Section 2, the case of instantaneous nonlinearity is considered, while the results taking into account the finite relaxation times of nonlinearity (noninstantaneous nonlinearity) are discussed in Section 3. Separate Section 4 is devoted to the self-trapping effect in the relaxing-nonlinearity case. The paper is closed with a brief conclusion.

## 2. Instantaneous nonlinearity

The basis of our calculations is the one-dimensional wave equation,

$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 (n^2 E)}{\partial t^2} = 0, \quad (1)$$

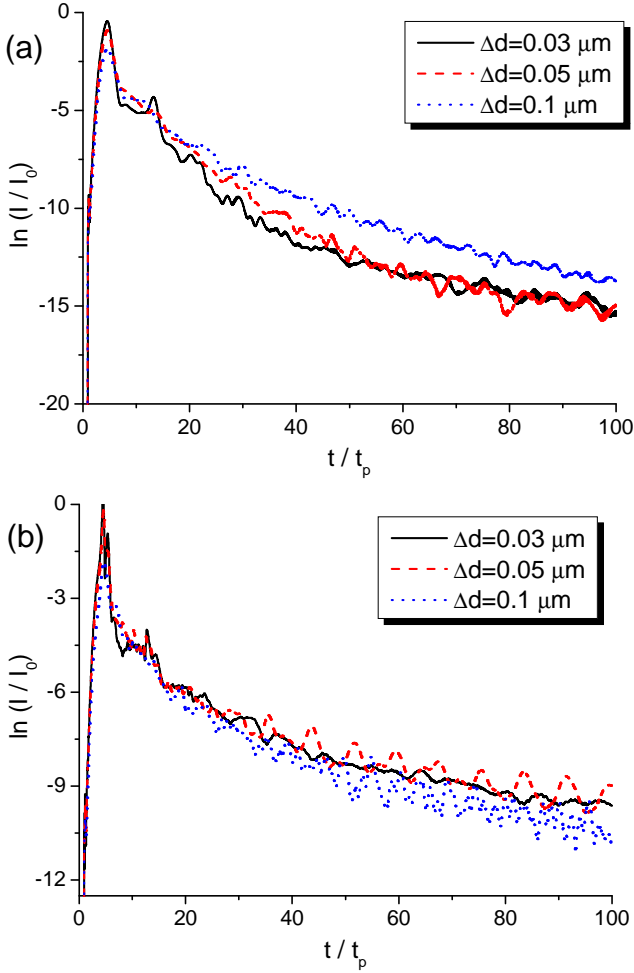


Fig. 3. (Color online) Dynamics of transmitted intensity logarithm for the photonic crystal with 50 periods at the nonlinearity coefficients (a)  $n_2 I_0 = 0.01$ , (b)  $n_2 I_0 = 0.05$ . The curves are averaged over 25 realizations.

where  $E$  is the electric field strength,  $n$  is the medium refractive index. In nonlinear medium, the latter depends on light intensity  $I = |E|^2$  as

$$n = n^0(z) + \delta n(I, t, z), \quad (2)$$

where  $n^0(z)$  is a linear part of refractive index. The  $z$ -dependence marks the periodic modulation inherent for the photonic crystal which, in our case, is a multilayer structure consisting of two different materials – layers  $a$  and  $b$ . In general, the nonlinear term  $\delta n$  takes into account the relaxation process which we describe with the Debye model of nonlinearity [29]

$$t_{nl} \frac{d\delta n}{dt} + \delta n = n_2 I, \quad (3)$$

where  $n_2$  is the Kerr nonlinear coefficient, and  $t_{nl}$  is the relaxation time. The disorder is introduced into the periodic structure through random variations of thicknesses of layers  $a$  and  $b$ ,

$$d_{a,b} = d_{a,b}^0 + \Delta d(\xi - 1/2), \quad (4)$$

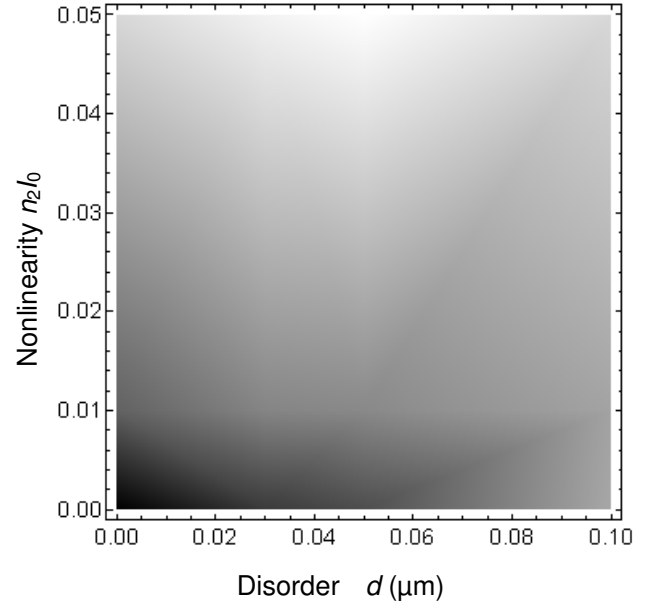


Fig. 4. The tail intensity deviation from the linear ordered case as a function of disorder and nonlinearity strengths. This density plot is based on the data of Figs. 2 and 3.

where  $d_{a,b}^0$  are the mean values of thicknesses,  $\Delta d$  is the amplitude of disorder, and  $\xi$  is the random quantity uniformly distributed in the range  $[0, 1]$ . To solve the equations listed above, we use the numerical approach developed in one of our previous publications [30].

For our model calculations, we adopt the parameters of the materials and structure as follows:  $d_a^0 = 0.4$  and  $d_b^0 = 0.24 \mu\text{m}$ ,  $n_a^0 = 2$  and  $n_b^0 = 1.5$ . It is worth to note that we do not mean any specific materials. The main feature of the system considered is the band gap in optical frequency region which requires layer thicknesses to be comparable to the wavelength and large enough difference between refractive indices. The interaction of radiation with the structure depends on the position of light frequency against the zone spectrum of the photonic crystal. The change in these parameters, obviously, will change the position and widths of the band gaps, so that one would have to make just the proper frequency tuning. We suppose that it will not lead to any qualitative changes in the light-matter interaction dynamics. In this section, we assume that the nonlinearity is instantaneous, i.e.  $t_{nl} = 0$ . The envelope of the pulse at the input of the photonic structure has Gaussian shape,  $A(t) = A_0 \exp(-t^2/2t_p^2)$ , where  $t_p$  is the pulse duration, and  $A_0$  is the amplitude of the radiation field. Let us illustrate influence of disorder on the properties of photonic crystal with Fig. 1 where the transmission spectra of the structure with  $N = 10$  periods are depicted. It is seen that strong disorder results in decrease of transmission outside the initial band gap. This can be interpreted as effective widening of the band gap in accordance with Ref. [28]. The waves attenuate exponentially while propagating in this widened forbid-

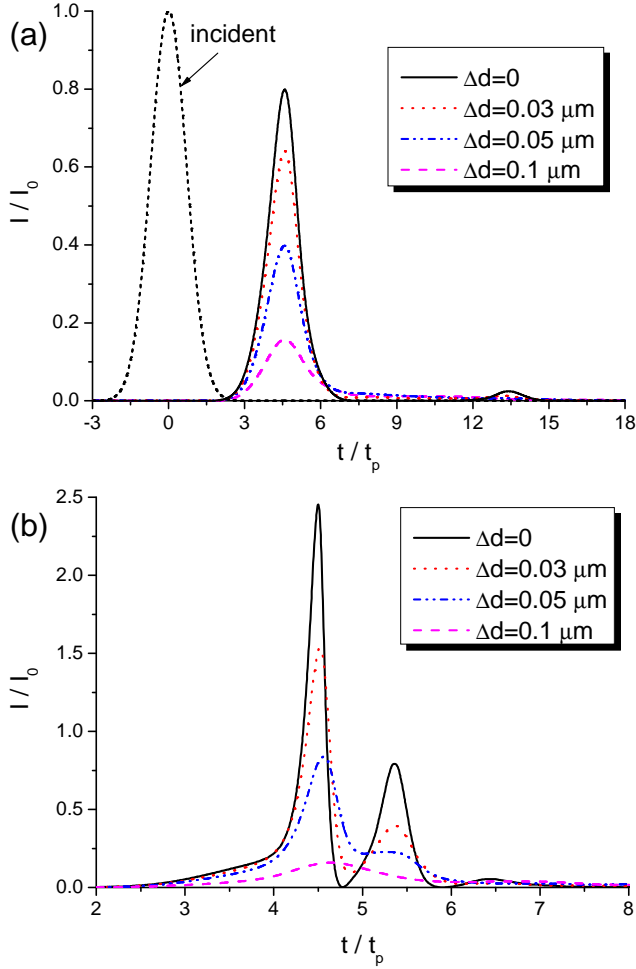


Fig. 5. (Color online) The profiles of transmitted pulses for the photonic crystal with 50 periods at the nonlinearity coefficients (a)  $n_2 I_0 = 0.01$ , (b)  $n_2 I_0 = 0.05$ . The curves are averaged over 25 realizations.

den region which is the feature of localization effect. For definiteness, we assume  $t_p = 50$  fs and the central wavelength  $\lambda_c = 1.064 \mu\text{m}$  in further calculations, i.e. the carrier frequency lies just outside the band gap of the perfect multilayer.

In this paper, we restrict ourselves to the structure with nonlinear  $b$  layers (second layers of the period), since light concentrates in these layers when we deal with the high-frequency edge of the band gap. There is a number of studies devoted to such semi-linear structures which can be used, for example, for optical nonlinearity management [31]. The strength of nonlinearity ( $n_2 I_0 = n_2 |A_0|^2 \sim 0.01$ ) is taken to be large enough to strongly influence the pulse characteristics. In addition, it allows to consider comparatively short systems, in this section we assume  $N = 50$  periods. Thus, we are interested in interplay of strong disorder and strong nonlinearity. This interplay can be studied on the short timescale (pulse shape transformation) and at long times (pulse tail transformation).

Let us start with the change of pulse decay pattern in the nonlinear structure. As was mentioned above, non-exponential behavior of the tail can be interpreted as the evidence of diffusion violation and the sign of the Anderson localization. In other words, the logarithm of intensity should be a linear function of time in the diffusion regime. As seen in Fig. 2, this dependence deviates from linear even in the linear case ( $n_2 = 0$ ), since there is no threshold for localization. Nonlinearity leads to stronger nonexponentiality which can be treated as light induced localization [17]. In other words, the number of photons traveling in the structure for a long time before eventually leaving it (long-lived localized states) is larger in the nonlinear case than in the linear one. Comparing Figs. 2(a) and (b), one can say that the larger disorder, the stronger nonlinearity is needed to observe the deviation from the linear tail. Figure 3 shows that the opposite is also true: the large nonlinearity coefficient implies the necessity to make disorder stronger than at lower nonlinearities to obtain the deviation from the nonlinear ordered system behavior. It is seen that in the case of  $n_2 I_0 = 0.05$ , the rise of disorder almost has no influence on the rate of intensity decay. Thus, nonlinearity and disorder seem to be the *competing* factors: strong disorder suppresses the manifestations of nonlinearity at large times, and vice versa. On the other hand, both factors act in the *same* direction: they tend to increase the nonexponentiality of the tail.

The results of Figs. 2 and 3 are summed up in Fig. 4: the change of grey level from black to white shows the deviation of the tail intensity (at  $t = 100t_p$ ) from the linear ordered case. It is seen that there is optimal value of disorder ( $\Delta d = 0.05 \mu\text{m}$ ) at which the influence of the strong nonlinearity on the pulse dynamics is maximal. When both these factors are very strong (right upper corner of the diagram), the resulting effect is not so pronounced. It is worth to emphasize that here we compared the results with the single linear ordered case, while in Figs. 2 and 3 we considered the deviations from the behavior of the corresponding linear disordered and nonlinear ordered systems.

Let us now proceed from the long timescale where the effects like localization can appear to short times and study the change of pulse form under joint influence of nonlinearity and disorder. The results of calculations for the two values of nonlinearity are demonstrated in Fig. 5. In the case of smaller value of nonlinearity [Fig. 5(a)], disorder results only in transmission attenuation and intensification of reflection. The peak of the pulse propagates with the same speed for all values of  $\Delta d$ . Strong nonlinearity is the reason of such effects as compression and dissociation of the pulse [Fig. 5(b)]. Disorder smoothes out and suppresses these effects, though at the smaller values of  $\Delta d$  the transmitted pulse is still shorter than the initial one. As the disorder parameter grows, the shape of the pulse tends to be more and more symmetric. Moreover, the peak of the transmitted pulse shifts to longer times. In other words, strong nonlin-

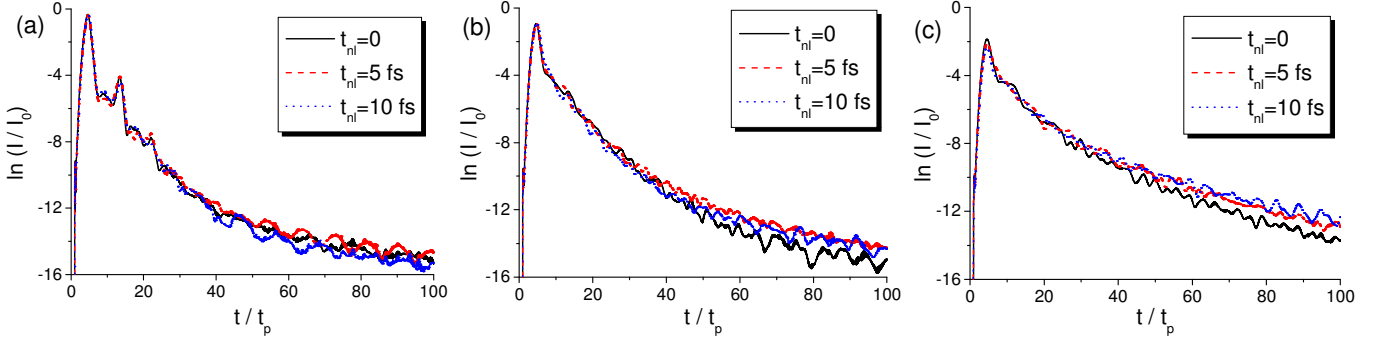


Fig. 6. (Color online) Dynamics of transmitted intensity logarithm for the photonic crystal with 50 periods at different relaxation times. The disorder amplitudes are (a)  $\Delta d = 0.02 \mu\text{m}$ , (b)  $\Delta d = 0.05 \mu\text{m}$ , (c)  $\Delta d = 0.1 \mu\text{m}$ . The nonlinearity coefficient is  $n_2 I_0 = 0.01$ . The curves are averaged over 25 realizations.

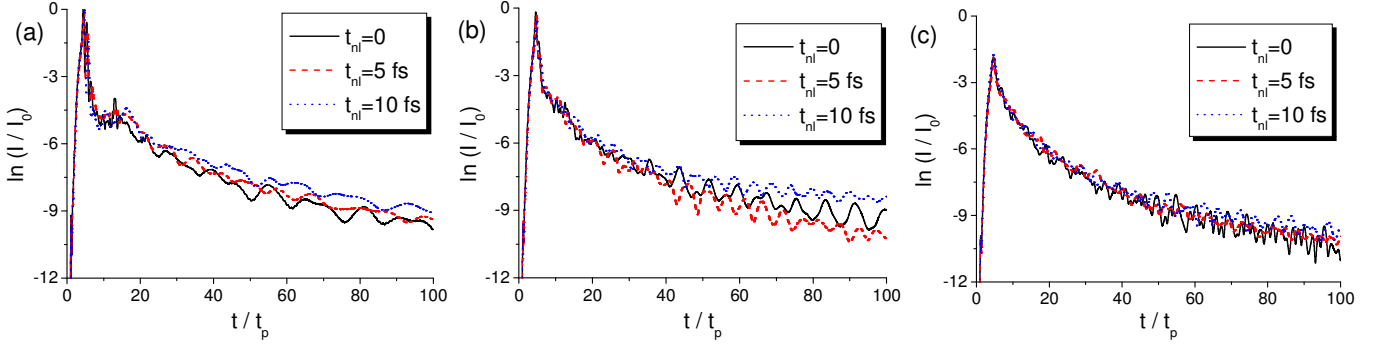


Fig. 7. (Color online) The same as in Fig. 6 but for the nonlinearity coefficient is  $n_2 I_0 = 0.05$ .

earity and disorder act *together* to slow down the pulse. Thus, both on the long and short timescale, the disorder and nonlinearity are the factors acting in the same direction in one respect and in the opposite directions in another respect.

### 3. Relaxing nonlinearity

Now let us turn to the noninstantaneous nonlinearity assuming that the relaxation time in Eq. (3) is not zero. In our calculations, the values  $t_{nl} = 5$  and  $10$  fs are used which approximately correspond to the electronic mechanism of relaxation; the pulse duration is still  $50$  fs. First, we discuss the results for the long timescale, i.e. the change of behavior of the tail due to the appearance of the relaxation. The case of relatively weak nonlinearity is shown in Fig. 6. Panel (a) of this figure demonstrates that, for both weak disorder and weak nonlinearity, the temporal dependence of the tail is unaffected by the relaxation process. Increasing disorder strength results in divergence of the curves: nonexponentiality of the tail increases for nonzero  $t_{nl}$  [see Fig. 6(b) and (c)]. This can be considered as indication of stronger Anderson localization for relaxing weak nonlinearity at large disorder parameters.

The results for strong nonlinearity are depicted in Fig. 7. This figure implies that the increase of disorder has the opposite effect: the curves for different relaxation

times take on similar behavior for large  $\Delta d$  [Fig. 7(c)], while for weak disorder they follow substantially different paths [Fig. 7(a) and (b)]. Thus, in the cases when both disorder and nonlinearity are strong or weak, the influence of relaxation is negligible on the long timescale. To see the change of behavior due to  $t_{nl} \neq 0$ , one should take either strong disorder and weak nonlinearity or vice versa.

On the short timescale, we are interested in the change of the peak intensity of the pulse under simultaneous presence of noninstantaneous nonlinearity and disorder. As pulse profiles in Figs. 8 and 9 show, the relationship between the factors is the same as on the long timescale. The influence of relaxation is minimal when both nonlinearity and disorder are weak [Fig. 8(a)]. The effect of relaxation is maximal when one of the factors is strong while the other is weak [Fig. 8(b) and 9(a)]. Finally, when both nonlinearity and disorder are strong, the change in pulse profiles is not great again [Fig. 9(b)], though in this case increase of relaxation time can even result in the rise of peak intensity (see the curve for  $t_{nl} = 10$  fs).

### 4. Self-trapping

In this section, we study the influence of disorder on the self-trapping of the pulse in the nonlinear photonic crystal. Self-trapping occurs when both the pulse duration



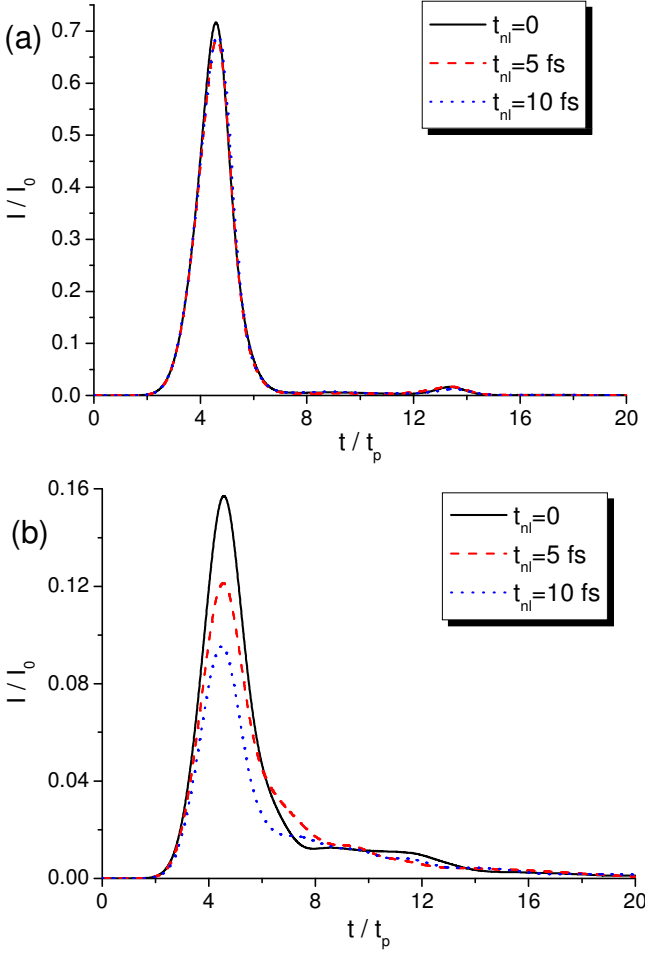


Fig. 8. (Color online) Profiles of the pulses transmitted through the photonic crystal with 50 periods at different relaxation times. The disorder amplitudes are (a)  $\Delta d = 0.02 \mu\text{m}$ , (b)  $\Delta d = 0.1 \mu\text{m}$ . The nonlinearity coefficient is  $n_2 I_0 = 0.01$ . The curves are averaged over 25 realizations.

and the relaxation time of nonlinearity are short enough and comparable with each other. The reason is the formation of a peculiar dynamical cavity (distribution of nonlinear index of refraction) which is stable enough to trap the energy of the pulse for a long time. Different aspects of the self-trapping effect were studied in our previous papers [30, 32–35]. To obtain self-trapping, in this section we assume shorter pulse ( $t_p = 30$  fs) and longer photonic crystal ( $N = 200$  periods). As previously, the half of the layers is linear, while the other half has relaxing nonlinearity. First, we assume the disorder is absent. The profiles of the transmitted and reflected radiation demonstrating self-trapping at  $t_{nl} = 10$  fs are shown in Fig. 10(a). One can easily see that the energy of the output radiation in this case is much smaller than the input energy, i.e. the most part of light is stored inside the structure. The distribution of radiation intensity over the system at the time instant  $t = 100t_p$  (more than three times larger than the propagation time of

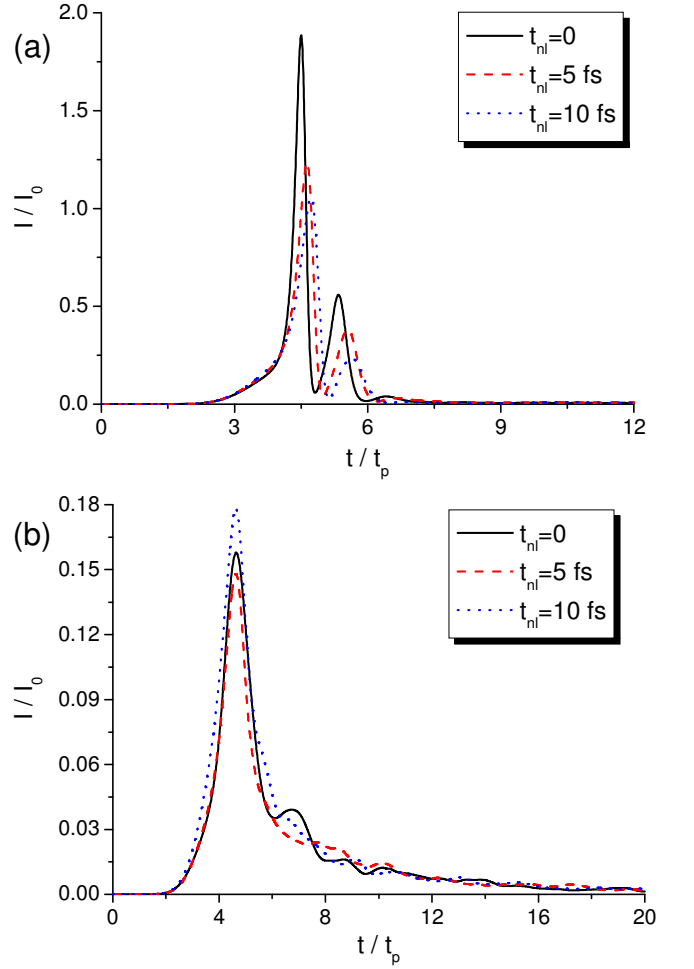


Fig. 9. (Color online) The same as in Fig. 8 but for the nonlinearity coefficient is  $n_2 I_0 = 0.05$ .

the pulse in the linear case) has the characteristic bell-shaped form resembling the envelope of the pulse [see Fig. 10(b)].

Now we can add the disorder. The dependence of the output energy integrated for the time  $100t_p$  on the disorder strength is shown in Fig. 11. It is seen that the dependence can be divided into three regions:

(i) the region of rapid rising where both the transmitted and reflected energies increase. This region occupies the disorder strengths from  $\Delta d = 0$  [order, when the transmission is low and the reflection is minimal, see Fig. 10(a)] to  $\Delta d = 0.02 \mu\text{m}$  [transmission has maximal value, see the profiles in Fig. 10(c)];

(ii) the region where the transmission curve rapidly decreases while the reflection curve has a depression. This region is between  $\Delta d = 0.02$  and  $0.05 \mu\text{m}$ ;

(iii) the region of gently sloping curves for  $\Delta d > 0.05 \mu\text{m}$  where transmission is negligible and reflection is very strong [see the profiles in Fig. 10(d)].

Thus, the main effect of disorder is the gradual increase of reflection, while the transmission has the maximum at a certain level of disorder. This behavior can

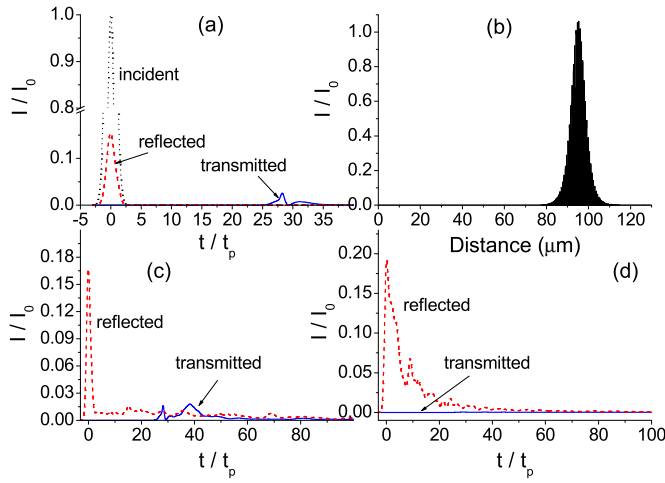


Fig. 10. (Color online) (a), (c), (d) Profiles of the transmitted and reflected radiation when the disorder amplitudes are  $\Delta d = 0, 0.02$  and  $0.1 \mu\text{m}$ , respectively. (b) The distribution of light intensity inside the structure at the time instant  $t = 100t_p$  in the ordered case. The photonic crystal has 200 periods (length  $128 \mu\text{m}$ ); the nonlinearity coefficient is  $n_2 I_0 = 0.01$  and the relaxation time is  $t_{nl} = 10$  fs. The curves in panels (c) and (d) are averaged over 25 realizations.

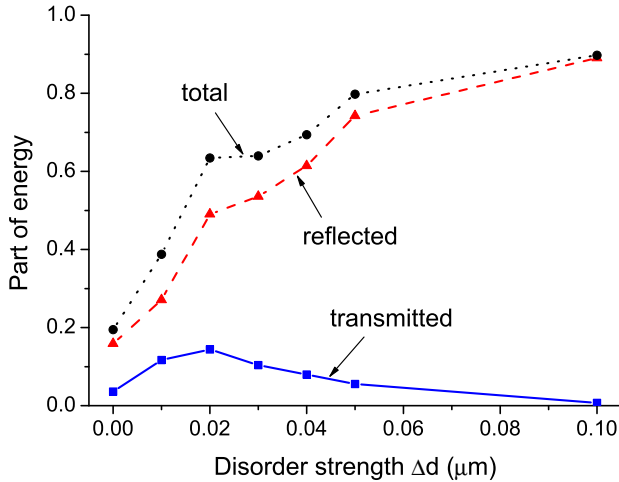


Fig. 11. (Color online) The dependence of the output energy (as a part of the input one) on the disorder strength. The curves are the result of averaging over 25 realizations.

be explained as follows. First, at low disorder strengths ( $\Delta d < 0.02 \mu\text{m}$ ), the dynamical trap (or cavity) loses its stability and more radiation leaves the structure during the same time. As a result, both the transmission and reflection grow. However, more than a half of light energy stays inside the system. Figure 12 shows four typical realizations of light distribution inside the disordered photonic crystal at the point of maximal transmission ( $\Delta d = 0.02 \mu\text{m}$ ). Rare realizations which appear once in the full set of realizations have a single high-intensity peak corresponding to the trap [Fig. 12(a)] or

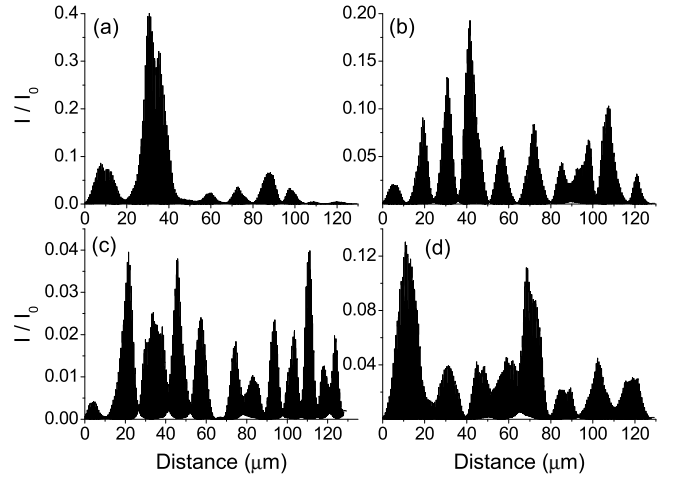


Fig. 12. Four realizations of the distribution of light intensity inside the structure at the time instant  $t = 100t_p$  in the case  $\Delta d = 0.02 \mu\text{m}$ .

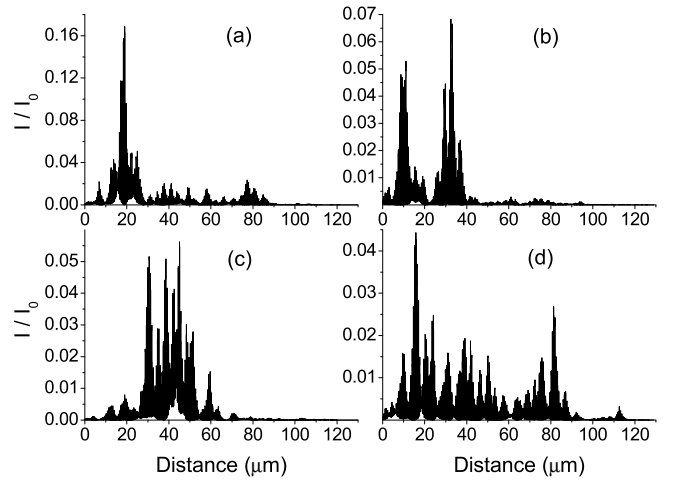


Fig. 13. The same as in Fig. 12 but for the disorder strength  $\Delta d = 0.1 \mu\text{m}$ .

many low-intensity peaks distributed over the length of the structure [Fig. 12(c)]. Between these extremes are the cases of many narrow peaks [Fig. 12(b)] or several wide peaks of intermediate intensity [Fig. 12(d)]. It is worth to note that the high-intensity sites of the distribution are concentrated mainly close to the input of the system, in contrast to the ordered case [compare with Fig. 10(b)].

For higher disorder strengths ( $\Delta d > 0.02 \mu\text{m}$ ), the process of reflection reinforcement typical for strongly disordered media becomes dominant. At the same time, transmission rapidly decays. As distribution realizations at  $\Delta d = 0.1 \mu\text{m}$  show (Fig. 13), the remanent radiation concentrates almost exclusively in the first half of the photonic crystal. Possible variants include a single narrow high-intensity peak near the input facet [Fig. 13(a)], two peaks of intermediate intensity [Fig. 13(b)], many

peaks in a narrow region [Fig. 13(c)], and, finally, a wide distribution of low-intensity radiation covering almost two third of the structure [Fig. 13(d)]. Thus, the maximum in the transmission curve of Fig. 11 is a result of two processes occurring due to disorder: reflection reinforcement and destruction of the trap.

## 5. Conclusion

In this paper, we analyzed the interplay of disorder and nonlinearity in several variants of one-dimensional multilayer structure. Calculations for both instantaneous and relaxing nonlinearity imply that nonlinearity and disorder are the competing factors: when one of them is strong, the influence of the other factor becomes apparent at substantially larger values than in the weak-first-factor case. In addition, in the regime of strong disorder and strong nonlinearity, the influence of relaxation appears to be negligible. Special attention was given to the modification of the self-trapping effect which still takes place for low enough disorder strengths. However, the patterns of light distribution along the photonic crystal are substantially changed as demonstrated by various realizations of disordered system. At higher disorders, the usual reflection prevails diminishing the part of energy trapped inside the structure.

Possible further directions of research include studies of spectral transformations and multi-pulse dynamics inside disordered photonic crystals. It is also important to increase the number of realizations. Though the qualitative conclusions based on the general trends of the curves seem to be reliable, the number of realizations needs to be improved in future.

Though the results reported in this paper were obtained for optical waves, we hope that they will be useful for the other physical situations as well, such as nonlinear acoustic waves, and are of interest from the wider perspective of general classical wave dynamics. Moreover, nonlinearity of the medium can be considered in some sense as an analogue of electron-electron interaction in the solid state disordered systems [36].

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